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Risk Finance and Asset Pricing

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Motivation and Book Introduction

Risk Finance and Engineering have been confronted in recent years with both immense challenges and opportunities. A challenge to bridge theory and practice following the important contributions made these last decades by Arrow and Debreu's fundamental theory of Assets Pricing. And its many uses to better comprehend the working of financial markets and price assets and their derivatives. A challenge to reconcile the doubts raised by assumptions of fundamental finance and opportunities to profit by the initiated who can appreciate the pro and cons of these theories. At both theoretical and practical levels finance theory has made extraordinary intellectual strides while contributing immensely to economic development. At the same time it has enriched the many financial engineers able to innovate and trade in financial products that create greater liquidity, predict and price assets, manage financial risks and contribute to the growth of financial markets.

The motivation for this book arose in the course of my lectures in the department of Finance and Risk Engineering at the New York University (NYU) Polytechnic Institute following the financial meltdown of 2008-2009. This was a year when risks and all their financial manifestations struck at the heart of financial citadels and world economies. No firm was "too big to fail" and risks hitherto conceived of theoretically, ignored or only dreamed of have revealed their potency. This was also a year when extreme events have come into their own: ex-ante ignored, but factual and painful ex-post for all those who ignored the unlikely. The whole world was hurting: unemployment, de-inflation of assets, and times of reckoning with greed, regulation, constraints, and finiteness of resources have become the underlying tune of financial discourse. Both persons and institutions have questioned the validity of financial models and their practical implications. On the academic front, challenging questions have been raised against the fundamental and complete markets dogma of finance, claiming that models can default and that incomplete markets are far more prevalent than theoretical finance would have us believe.

The financial meltdown of 2008-2009 has also ignited a far greater concern for the underlying purposes of finance, not only as a means to get rich but to confront the risks that beset us—whether predictable or not. These include a population growth, environmental challenges, a globalization of finance, infrastructure, wellness etc.. These are real problems of common and personal importance with a transparent finance being part of the answer—an answer as a means and not only for self serving ends. The intent of this book is to provide both an accessible formulation of theoretical financial constructs embedded in a broad variety of real and useful problems. 2008/2009 has revealed also that risks borne by the non-initiated to the complexity of financial products and markets can be very costly. It has also become apparent that corporations and financial firms, traditionally managing real resources have gradually shifted their economic activity by turning to financial manipulations, acting as intermediaries with losses assumed by non-informed investors. These firms have capitalized on leverage and short term returns and strapping healthy corporations with a debt they may not be able to bear. Governmental institutions have not been spared either. They too have turned to financial markets to seek the funds needed for investments in infrastructure or meet their financing needs. A "Pandora Financial Box" has been opened, and Finance, for all the good and the risks it deals with and manages, has also the potential to make great damage if not understood.

Further, there is an increased awareness that financial systems are changing. For example the traditional role of banks to provide liquidity to borrowers and business firms may have been jeopardized in their pursuit of (short term) profits. These financial institutions have become marketers of financial products and intermediaries to ever growing financial markets rather than be the providers of liquidity that underlie

their charter granted by society and its governments. In the pursuit of profits, new financial institutions and previously non-financial firms have emerged and converged in new enterprises that provide both financial services and manage their own economic interests. These firms, such as insurance companies provide liquidity and are transforming the financial system. In these processes, financial engineers remain the means to provide financial products, help decide how and where to invest and how to manage risks. The insurance-finance convergence has also provided means to assure buyers and sellers and thus contribute to the liquidity needed. The recently creation of a Global Insurance Exchange in New York to cover complex risks, modeled after Lloyd's of London (The New York Times, January, 2010, p. B3) is just such an example.

The book is intended for both beginning and practicing financial engineers, seeing to provide an appreciation and pricing of real financial problems. Throughout my classes I have become aware that many concepts transparent to mathematically savvy students are not understood by others. Inversely, many students with an extensive mathematics background fail to understand that financial engineering is not about mathematics but about complex relationships between buyers and sellers acting in financial markets imputing values and prices to “about everything that can be traded”. To better appreciate what financial engineering is, can do and its limitations, it is necessary to have a strong footing in principles of economics and finance, data and statistical analysis, personal utility and their behavioral manifestations in financial markets and financial modeling. In particular, financial modeling provides a means to interpret implied values and prices such as options, credit derivatives etc. In this sense, financial engineering is both real and virtual. Its usefulness is fueled by the needs of financial parties and by its potential contributions to investors, speculators and the society at large. The perspectives of this book, unlike many and important books in financial engineering and mathematics, are thus: to bridge theory and practice; to study financial engineering as a means and not only as an end to make money and emphasize a real finance to provide the support needed to meet both individual and collective needs. At the same time, the book emphasizes an intuitive and comprehensive approach to the foundations of risk finance and its many applications to assets pricing, real financial problems and financial risk management. In such a frame of mind, the theoretical frameworks of expected utility, Arrow-Debreu's foundations of fundamental finance, basic statistical manipulations of data and financial modeling, are both relevant and complementary.

Theoretical concepts and theories applied mindlessly can have dire consequences. Thus, understanding the underlying rationales that financial engineers use in financial modeling, optimization and decision making is important. By the same token, financial engineers cannot be “the canary in the coal mine” and ought to recognize that there is an inherent social and ethical responsibility that need not contradict the pursuit of wealth and money. There are as many opportunities to profit by contributing to economic sustainability, to investments in needed infrastructures, to preventing “booms and busts”, to reducing social inequities, to pointing to market potential defaults and failures etc. As there are opportunities: to profit from the design of complex and marketable financial products that provide greater and needed financial liquidity; to seek arbitrage opportunities and better forecast financial market prices.

The many applications treated in this book, drawn from a variety of financial, engineering and business professions includes insurance, pricing corporate loans and managing their risks, pricing safety and reliability, pricing franchises, operations risks, environmental quality and its control, infrastructure pricing, pricing “water”, pricing the insurance of rare events and uncommon risks etc. These applications are used to provide a motivation and a background for a greater appreciation of finance and its risk engineering.

The book is structured as follows. A first part consisting of chapters 1 and 2 provides an introduction to the business of finance, risk and their many applications. Issues such as ethics and finance are discussed. A second part, including chapters 3 and 4 are an introduction to risk measurement and to various

statistical approaches to do so. These chapters use data to measure risk, estimate financial trends, financial volatility and the many terms that make up the essential content of basic financial applications. These two chapters introduce the student to the need to confront the measurement, the “quantification of finance” and basic analyses using financial data. Chapter 4, is of a more advanced nature however and emphasize the problems of dependence including: statistical dependence, complexity, contagious risks, latent and black swan risks. The rationale for introducing these complex issues prior to a thorough study of financial and economic constructs used by financial engineers is to point out the true complexity of quant finance which cannot be explained always by available theories. Allowing students to grapple with complicated issues, sooner rather than later provide a challenge that is similar to the concerns and the manner in which we proceed to financial risk management.

A third part of the book includes Chapters 5 and 6. These chapters introduce the concept of utility and financial risk management. Many theories applied in financial economics are application or interpreted in terms of utility concepts. These include risk aversion, portfolio selection, certain equivalents in financial valuation, the CAPM, kernel pricing, insurance and utility-based risk management. These applications are still profusely used (explicitly or implicitly) many practical problems. The presumption that financial engineering is essentially concerned with options’ pricing is, I believe, misguided. These chapters will show through applications that underlying financial theory there are almost always three issues to reckon with: the rationality of the parties to a financial transaction, their private and common information and the market price. In many cases, any two would imply the other. In other words, any model in fundamental finance implies in fact an underlying-rationality—which when violated leads to model defaults.

The fourth part of the book includes chapters 7 and 8. These chapter outline the Arrow-Debreu framework in discrete states and time for assets and derivatives (options) pricing (chapter 7). An intuitive introduction to Martingales and their importance for asset pricing is included in the appendix to chapter 7. Chapter 8 provides a review of financial markets and optional portfolios used to manage and trade risks. These chapters present the basic concept of fundamental finance. The theory is discussed, criticized and applied to many examples. To keep this introduction tractable (without losing its essential implications and applications) simple binomial, multinomial and discrete state models are used. Extensions to continuous time finance are considered briefly with specific chapters to be posted on the book website www.charlestapiero.com. Applications to a variety of problems including derivatives pricing, default bonds, pricing insurance contracts, stochastic volatility models, multiple sources of risks models and a plethora of problems commonly treated in practice and in advanced texts are also presented and solved. Throughout these chapters, issues and instruments of current interest, such as the financial meltdown of 2008, volatility and chaos, globalization, outsourcing etc. are used to explain these important facets of financial practice and the limits of the current theoretical models of finance.

Finally, the last part of the book consists of chapters 9, 10 and 11. Chapters 9 and 10 deal with credit risk and scoring, multi names credit risk and credit derivatives. Several approaches to pricing credit risk are outlined. Following the credit crisis, a greater awareness has set in that these risks ought to be better regulated. Chapter 10 focuses on multi names credit risk portfolios, and structured financial products such as CDO, CMO, CLO etc. A last chapter 11 addresses the important and practical problems calculating an implied volatility and an implied risk neutral distribution. Three approaches are emphasized: parametric, a-parametric and a utility-rationality based approach. These three chapters can be seen as a whole that can be delivered as one course on Credit Risk.

This book is the product of my lectures and research. I have used knowingly and unknowingly others’ contributions, or interpreted their contributions in a manner that would be coherent with the intent of this book. I apologize if I have failed to refer to some of their work.

Further, discussions with well known financial academics and managers have contributed to these volumes. Among the many I have consulted, are included Nassim Taleb and his extraordinary efforts to remind us all that finance is about practice—not only theories, Alain Bensoussan, Pierre Vallois, Tyrone Duncan, Elias Shiu, Lorne Switzer, Sebatstien Galy, Daniel Totouom-Tangho, Jean Carlo Bonila, Roy Freedman, Daniel, Dafna and Oren Tapiero (each of whom is involved in the real world of finance and economic development), Konstantin Kogan, Aime Scannavino, Bertrand Munier, Mirela Ivan, Kevin Koshy and many others whose names will fill pages of this book. Finally, this book could not be written without the tolerance and the love of my life partner Carole Solomon , a woman of Great Distinction who had to endure the many moments that required the bubbles that I had to escape into in order to achieve this book.

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